Hadrian Park Primary School
Bar Modelling

Sept 2022

Contents

Introduction

Gu (2015) "If we don't use concrete manipulations, we cannot understand the mathematics. If we just use manipulations, we aren't actually doing the mathematics."

Bar modelling is a useful tool to use across all areas of mathematics. It gives children a flexible and robust tool in their toolkits which they can they use to solve different mathematical problems in different contexts.

Bar modelling is not just about drawing diagrams. When we teach children to use bar models in the context of solving mathematical problems, they should always be recording the calculations that these are telling them to use.

Much of the bar modelling done in EYFS and KS1 will parallel the types of bar models required to work out how to solve complex KS2 and GCSE level questions, so it is important that these skills are taught consistently throughout a child's time at primary school.

The objectives and examples shown for each year group are the minimum we would expect children to be able to use by the end of each year. It may be appropriate for some children to access more sophisticated models from higher year groups, but it is necessary to keep the mathematical content within the year group they are in.

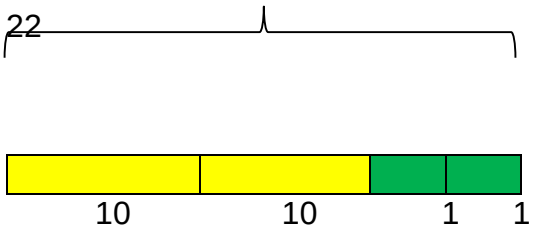
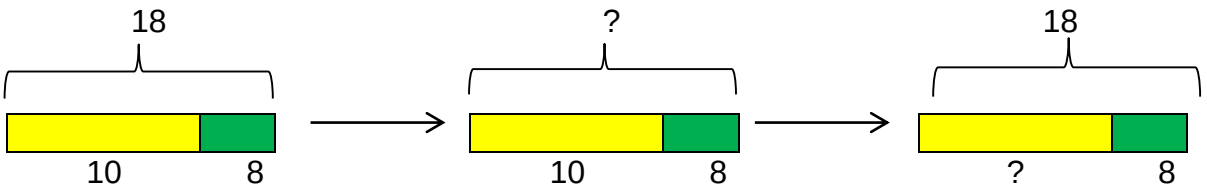
Tips for using bar models

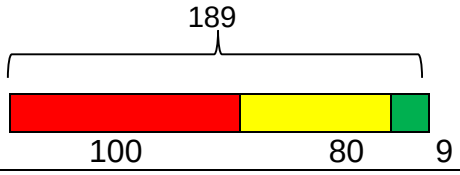
No bar model should ever be described as 'wrong' as long as it is accurately reflecting the mathematics that the children need to do and has helped them to access the calculations they need to solve. However, during early implementation, it will be necessary to guide children towards specific types of bar modelling until this becomes a flexible tool for them.

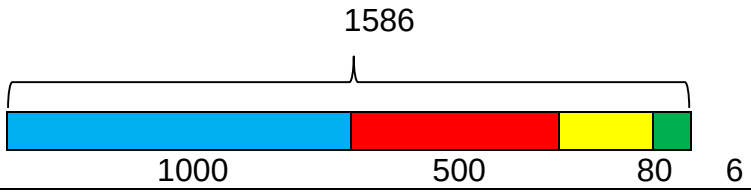
Children should be encouraged to make the bar models work for them. It does not matter whether we write on the bars or not. The advantage of not writing on the bars and using the brackets throughout is it allows children to get into the habit of manipulating and writing on their bars to adapt them as appropriate to the question.

Ensure when drawing that children are using the squares in their maths book so that the bars are an accurate size. It may be more appropriate and is acceptable at times to allow children to draw their bars on plain paper or even for adults to provide children with pre-drawn bars.

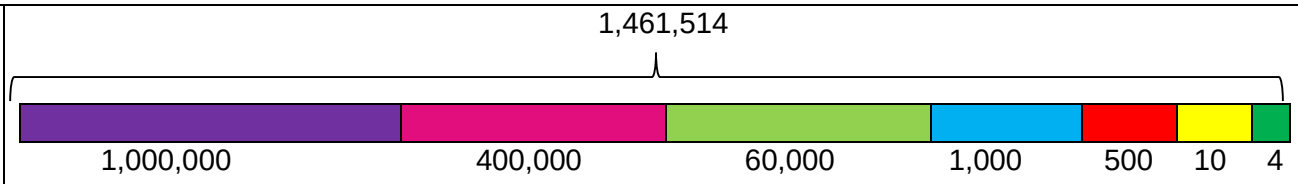
Place Value

Year 2	• Recognise the place value of each digit in a two-digit number (tens, ones) Pictorial representation of discrete bar model where each box represents one whole (Year 1). ↓ Concrete representation of continuous bar model where each strip of paper represents a number ↓ Pictorial representation of continuous bar model where each box represents a number.
	 Each ten and one represented with own section of the bar. Link to use of Dienes/Place value counters. Move on to combining all tens together and all ones together. 

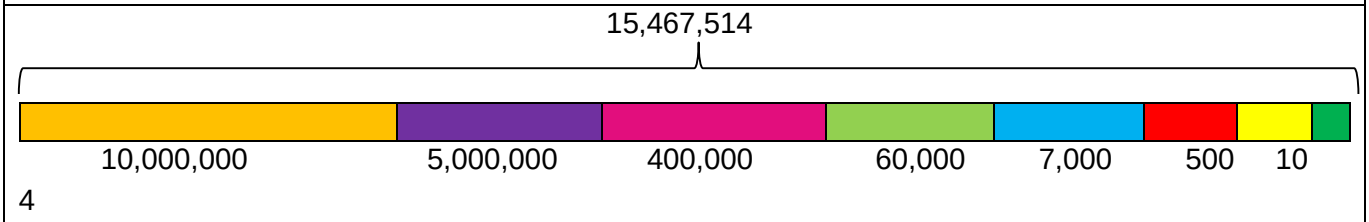
Year 3	• Recognise the place value of each digit in a three-digit number (hundreds, tens, ones) Concrete representation of continuous bar model where each strip of paper represents a number ↓ Pictorial representation of continuous bar model where each box represents a number.
	

Year 4	• Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones) Concrete representation of continuous bar model where each strip of paper represents a number ↓ Pictorial representation of continuous bar model where each box represents a number.
	

Year 5	• Read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit Pictorial representation of continuous bar model where each box represents a number.



Year 6	• Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit. Pictorial representation of continuous bar model where each box represents a number.



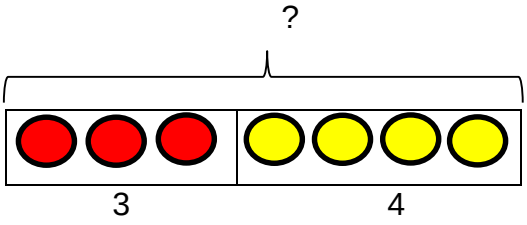
Please note the colour key for partitioning:

1s	
10s	
100s	
1000s	
10,000s	
100,000s	
1,000,000s	
10,000,000s	

Addition and Subtraction

EYFS	• Using quantities and objects, they add and subtract 2 single-digit numbers and count on or back to find the answer Part-part whole model using physical objects given in a question (e.g. cars, teddies) and placing in a pre-drawn bar provided by an adult.

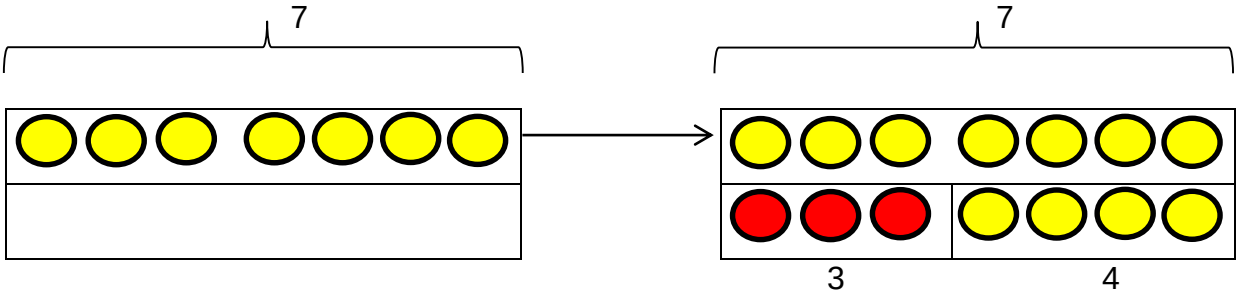
3 + 4 = ?



Children can count the total number of counters using 1:1 correspondence.

7 – 3 = ?

Swap 3 objects over for another colour
or cover them up
or take them out of the bar to see what is left



Children should be able to flexible partition numbers to represent them in different ways. E.g.) 7 = 3 + 4 but also 7 = 5 + 2. This can be shown with the use of multiple bar models.

Year 1

- Represent and use number bonds and related subtraction facts within 20
- Add and subtract one-digit and two-digit numbers to 20, including zero
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = □ – 9.

Part-part whole model using physical objects with 1:1 correspondence with the actual object in the question

↓

Part-part whole model using physical objects with 1:1 correspondence (e.g. different coloured counters, multilink, stones)

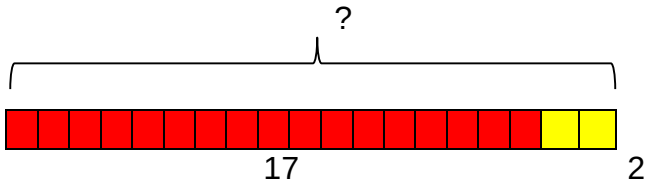
↓

Pictorial representation of discrete bar model where each box represents one whole.

<u>Aggregation (adding 2 separate parts together)</u> <u>*Using discrete bar models (each 'one' represented by an actual box).*</u>

17 + 2 = ?

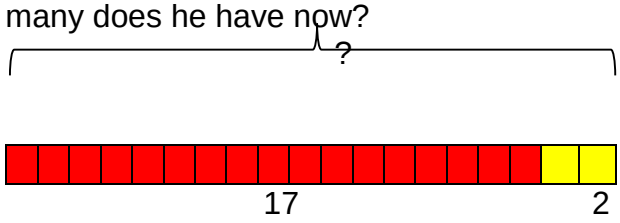
Peter has 17 cars and Jane has 2. How many do they have altogether?



<u>Augmentation (adding on to one single quantity)</u>
<u>*Using discrete bar models (each 'one' represented by an actual box).*</u>

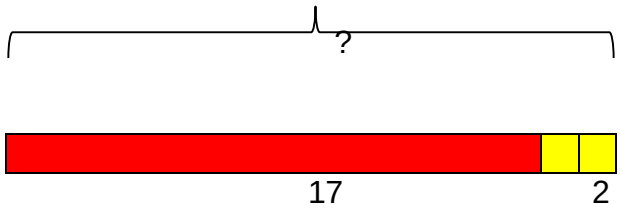
17 + 2 = ?

Peter has 17 cars. He buys 2 more. How many does he have now?



more

Progress to mixed bar modelling for these two types of addition modelling



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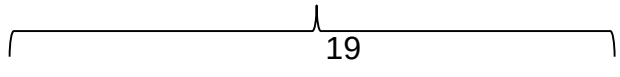
<u>Missing number addition (counting on)</u>

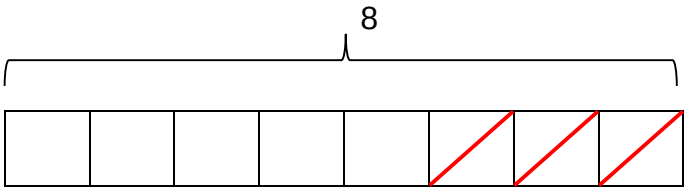
17 + ? = 19



<u>Subtraction (taking away)</u>

8 – 3 = ?

Jane has 8 apples. She eats 3. How many is she left with?

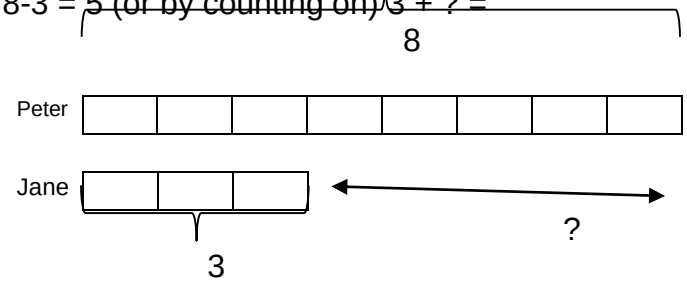


eaten

<u>Subtraction – difference</u>

Peter has 8 apples. Jane has 3. How many more apples does Peter have?

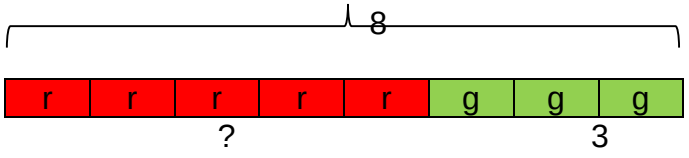
8-3 = 5 (or by counting on) 3 + ? = 8



<u>Subtraction (reduction)</u>

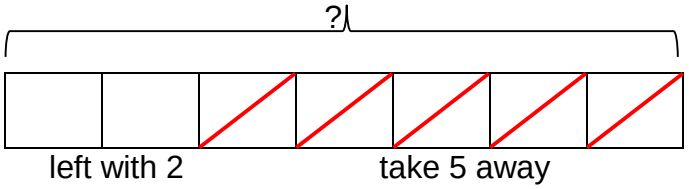
8 – 3 = ?

Jane has 8 apples. 3 are green. How many are red?

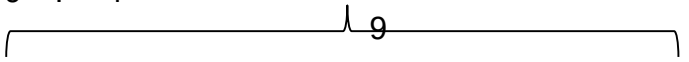


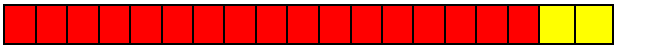
<u>Missing number subtraction</u>

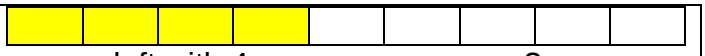
2 = ? – 5



9 - ? = 4



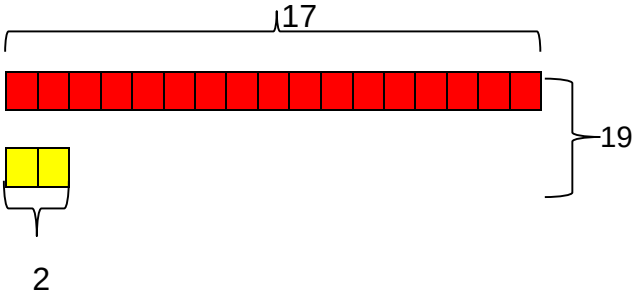




Comparison model for addition

Addition can also be shown using a comparison model:

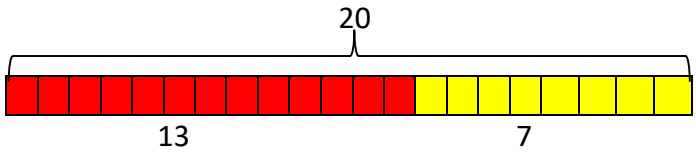
17 + 2 = 19



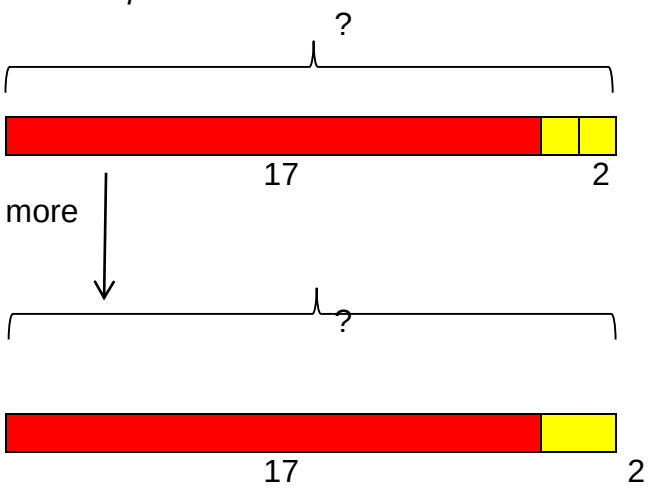
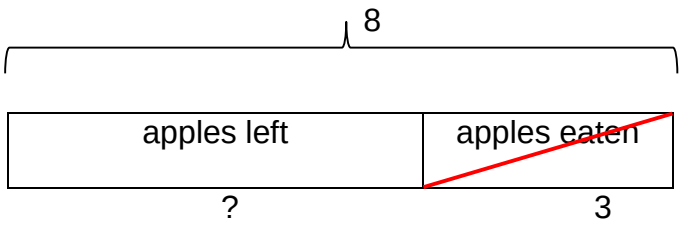
Fact Families for Number Bonds to 20

Using an addition addition bar model, children can represent fact families for number bonds to 20.

7 + 13 = 20
13 + 7 = 20
20 – 7 = 13
20 – 13 = 7



Year 2	• solve problems with addition and subtraction:
	  ○ using concrete objects and pictorial representations, including those involving numbers, quantities and measures
	  ○ applying their increasing knowledge of mental and written methods
	• recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
	• add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
	  ○ a two-digit number and ones
	  ○ a two-digit number and tens
	  ○ two two-digit numbers
	  ○ adding three one-digit numbers
	• show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
	• recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.
	
	Pictorial representation of discrete bar model where each box represents one whole (Year 1).
	↓
	Concrete representation of continuous bar model where each strip of paper represents a number
	↓
	Pictorial representation of continuous bar model where each box represents a number.

	<u>Addition (2 digit and ones)</u> 17 + 2 = ? Peter has 17 cars. He buys 2 more. How many does he have now? *Mixed representation*  *Continuous representation*
	<u>Subtraction</u> Jane has 8 apples. She eats 3. How many are left? 8 – 3 = ? 

Addition (2 digit and tens)

Peter has £23. He gets £30 for his birthday. How much does he have altogether?

23 + 30 =

?

	£10	£10	£10

£23 £30

2 line model

Adding 3 (or more!) one-digit numbers

9 + 3 + 1 = ?

?

9 3 1

Missing Number Problems

At Rory's birthday party, 12 children had the choice of vanilla, chocolate or strawberry ice-cream. 4 children chose vanilla and three children chose chocolate. How many chose strawberry?

Draw what you know:

12

4 + 3 = 7 12 – 7 = 5 so 5 children chose strawberry

V	C	S
4	3	?

7

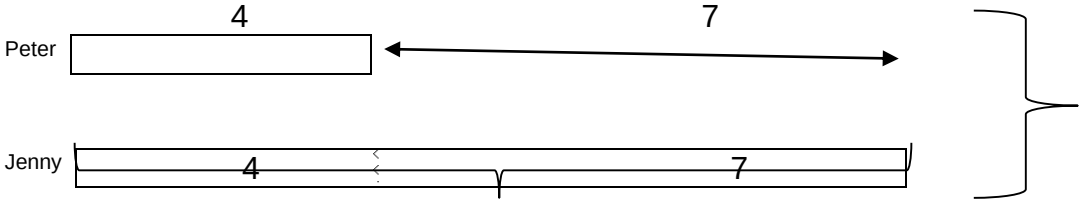
☐ - 14 = 32 14 + 32 = 46 so **46** – 14 = 32

?

What we are taking	What is left
14	32

Comparison problems

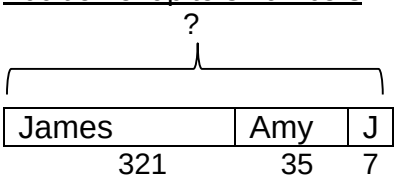
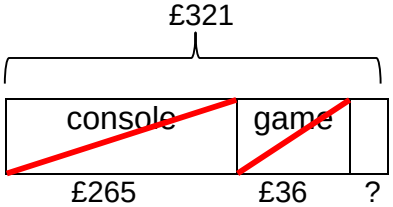
(e.g.) Peter has 4 sweets. Jenny has seven more sweets than Peter. How many sweets do they have altogether?



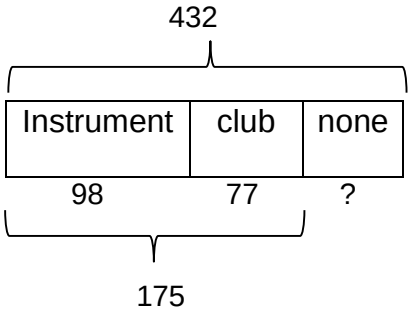
11 + 4 = 15

4 + 7 = 11

Year 3 and Year 4	Year 3
	• add and subtract numbers mentally, including:
	  ○ a three-digit number and ones
	  ○ a three-digit number and tens
	  ○ a three-digit number and hundreds
	• add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
	• estimate the answer to a calculation and use inverse operations to check answers
	• solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.
	
	Year 4
	• add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
	• estimate and use inverse operations to check answers to a calculation
	• solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.
	
	Concrete representation of continuous bar model where each strip of paper represents a number
	↓
	Pictorial representation of continuous bar model where each box represents a number.

	Methods continue as per Year 2 problems but with numbers up to 3 digits for Year 3 and 4 digits for Year 4. Focus on use for word problems. Addition of up to 3 numbers  James has 321 stickers, Amy has 35 and Jess has 7. How many do they have altogether? 321 + 35 + 7 = ☐ Subtraction  Ally has £321. She spends £265 on a new games console and £36 on a game. How much does she have left? 321 – 265 – 36 = ☐

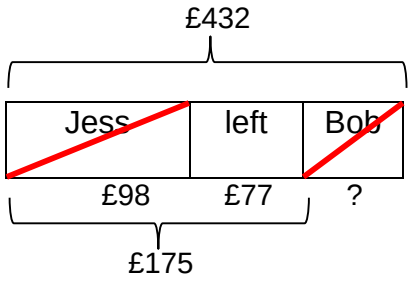
Missing numbers



There are 432 pupils in a school. 98 play an instrument and 77 take part in an after school club. How many children do not do either?

98 + 77 + ☐ = 432 so 432 – 175 = ☐

or

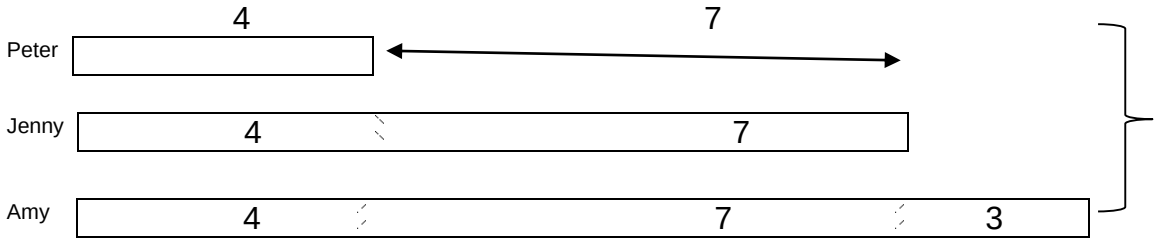


Amy had £432. She gave some to Bob, £98 to Jess and was left with £77. How much did she give to Bob?

98 + 77 + ☐ = 432 so 432 – 175 = ☐

Comparison problems

(e.g.) Peter has 4 sweets. Jenny has seven more sweets than Peter. Amy has 3 more sweets than Jenny.
How many sweets do they have altogether?

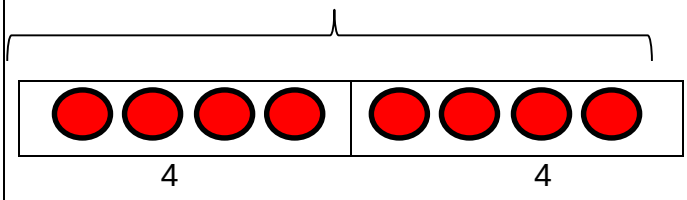


Year 5 and 6	• add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) • add and subtract numbers mentally with increasingly large numbers • use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why. Pictorial representation of continuous bar model where each box represents a number.
	Continue as per Years 3 and 4 to solve word problems and missing number problems with increasingly large numbers. Children move on to problems requiring them to calculate with more than 3 numbers. Bar modelling involving addition and subtraction in Y5/6 is likely to also require some modelling of multiplicative and divisive reasoning and fractions, decimals and percentages reasoning.

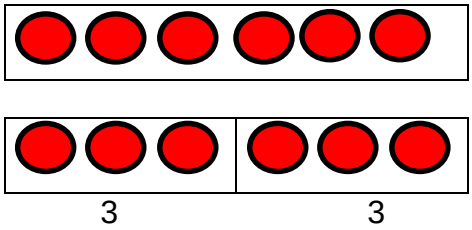
Multiplication and Division

<table>
<tr><td>EYFS</td><td colspan="2">Solve problems, including doubling, halving and sharing.

Part-part whole model using physical objects given in a question (e.g. cars, teddies) and placing in a pre-drawn bar provided by an adult.</td></tr>
<tr><td></td><td>Doubling

Double 4 ?
 

2 x 4 = 8</td><td>Halving and Sharing

Half of 6 6
 

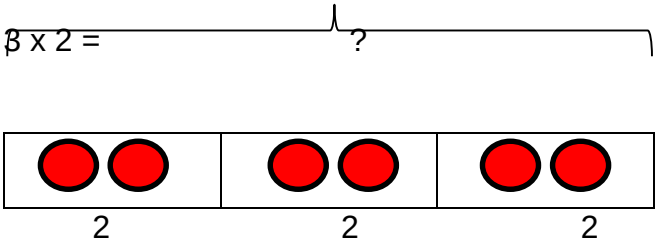
6 ÷ 2 = 3</td></tr>
</table>

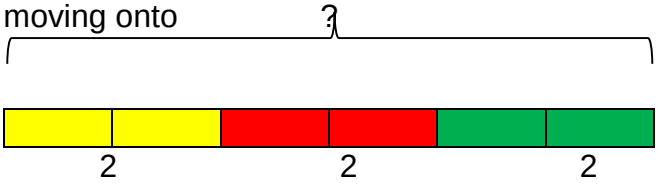
<table>
<tr><td>Year 1</td><td colspan="2">Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher

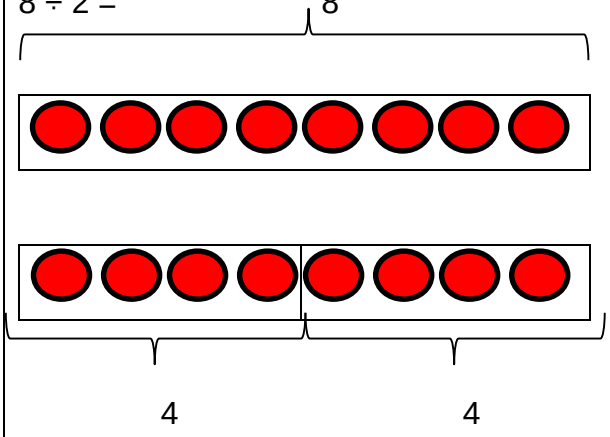
Part-part whole model using physical objects given in a question (e.g. cars, teddies) and placing in a pre-drawn bar provided by an adult.
↓

Using concrete objects with 1:1 correspondence (e.g. different coloured counters, multilink, stones)
↓

Pictorial representation of discrete bar model where each box represents one whole.</td></tr>
<tr><td></td><td>Multiplication

3 x 2 = ?
 

moving onto ?
 
</td><td>Division

8 ÷ 2 = 8
 
</td></tr>
</table>

Year 2

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Pictorial representation of discrete bar model where each box represents one whole (Year 1).

↓

Concrete representation of continuous bar model where each strip of paper represents a number

↓

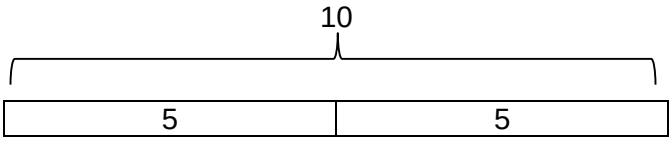
Pictorial representation of continuous bar model where each box represents a number.

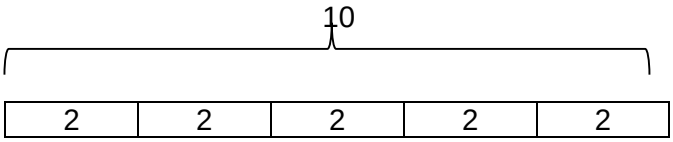
<u>Multiplication</u>

Models should be moving towards being continuous

Children apply comparison knowledge to solve missing number questions/balance equations

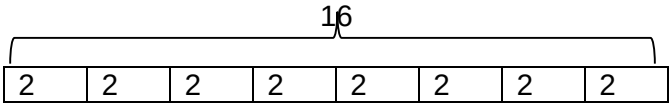
5 x 2 = 2 x 5



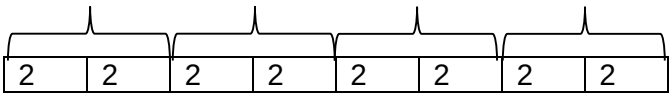


Children should be exploring different ways of partitioning their bar models.

E.g. 8 x 2 =



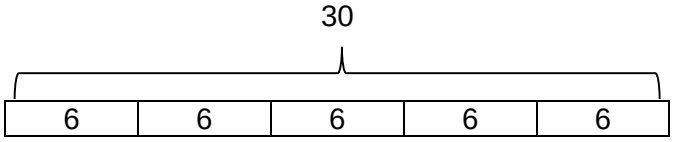
Is the same as
(2 x 2) + (2 x 2) + (2 x 2) + (2 x 2)



They should be exploring as many ways of adapting and partitioning these types of bars models as possible. This will bridge towards complex mental manipulations as they move through KS2. (e.g. looking at 32 x 25 as 8 x 4 x 25 rather than written)

<u>Division (sharing – partitive division)</u>

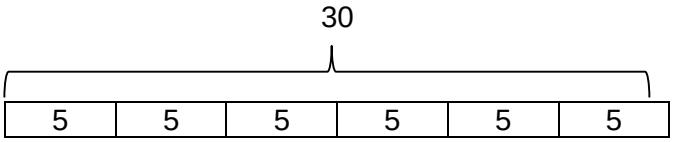
Jane has 30 cakes. She wants to share them equally between five boxes. How many should go in each box? 30 ÷ 5 = 6



<u>Division (grouping – quotative division)</u>

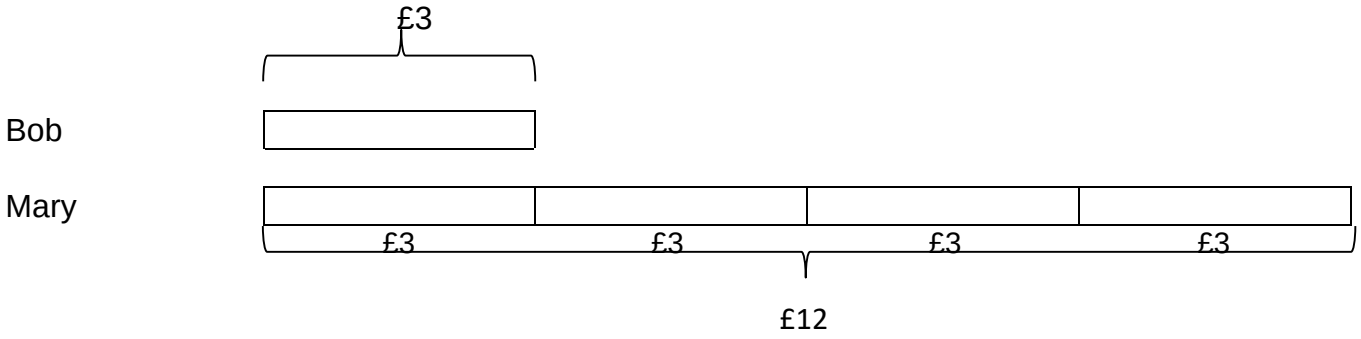
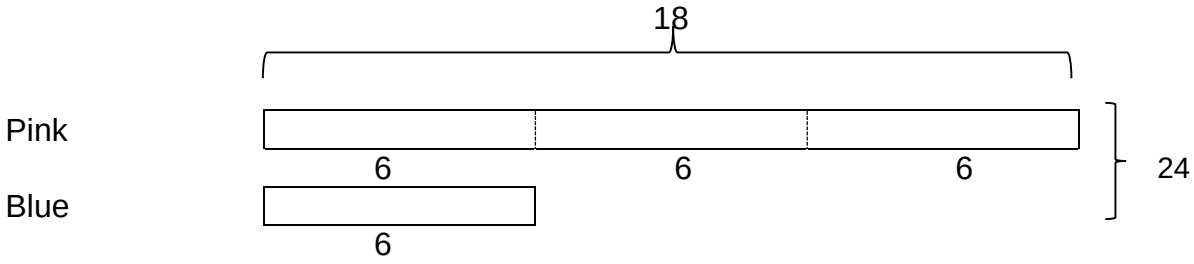
Jane has 30 cakes. She wants to pack them into boxes with 5 cakes in each box. How many **boxes** will she need? 30 ÷ 6 = 5

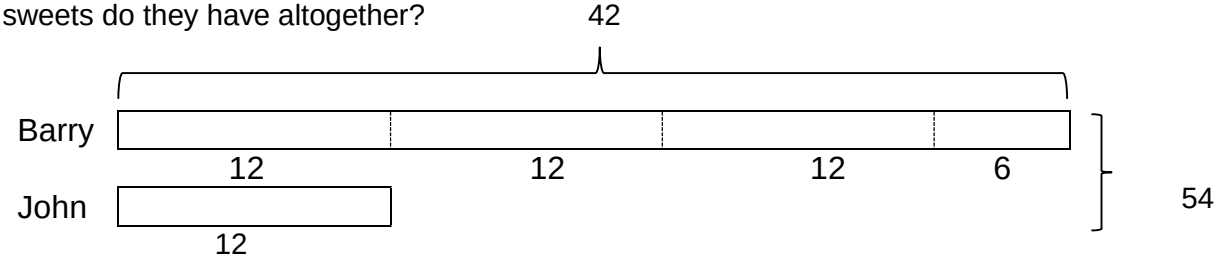
This model would be built up gradually, leaving the bar open ended and adding on boxes of 5 until 30 is reached.



Stem sentence you may wish to use:

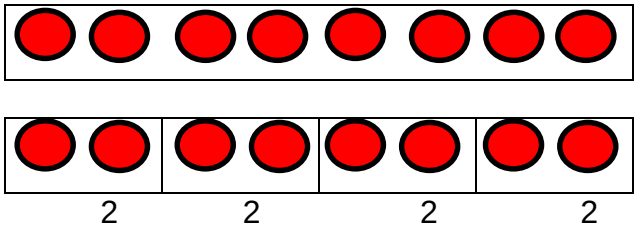
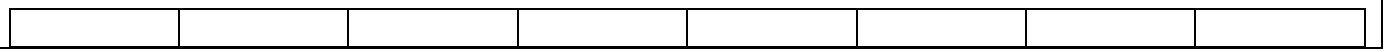
Number of boxes needed = ?

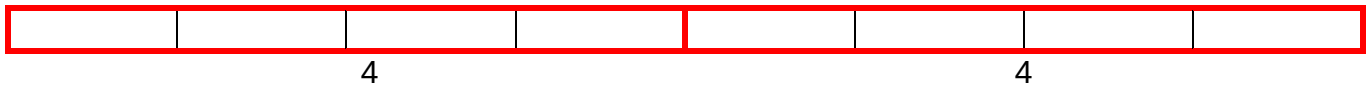
Years 3 and 4	<u>Year 3</u> • recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables • write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written method • solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects. <u>Year 4</u> • recall multiplication and division facts for multiplication tables up to 12 × 12 • use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers • recognise and use factor pairs and commutativity in mental calculations • multiply two-digit and three-digit numbers by a one-digit number using formal written layout • solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects. Concrete representation of continuous bar model where each strip of paper represents a number ↓ Pictorial representation of continuous bar model where each box represents a number.
	<u>Scaling/comparison</u> Bob has £3 in his piggy bank. Mary has 4 times as much as Bob. How much does Mary have?  All the girls in Libby's class have a pencil case. 18 girls have pink pencil cases. 3 times fewer girls have blue pencil cases. How many girls are there altogether? 

Year 5 and 6	Year 5 • identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers • know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers • establish whether a number up to 100 is prime and recall prime numbers up to 19 • multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers • multiply and divide numbers mentally drawing upon known facts • divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context • multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 • recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}) • solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes • solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign • solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates. Year 6 • multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication • divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context • divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context • perform mental calculations, including with mixed operations and large numbers Pictorial representation of continuous bar model where each box represents a number.
	Combining multiplication and division problems Fractional scaling Barry has 3 and a half times the amount of sweets that John has. If John has 12 sweets, how many sweets do they have altogether? 

Fractions

EYFS	Solve problems, including doubling, halving and sharing. Part-part whole model using physical objects (e.g. different coloured counters, multilink, stones) and placing in a pre-drawn bar provided by an adult.
	See multiplication and division

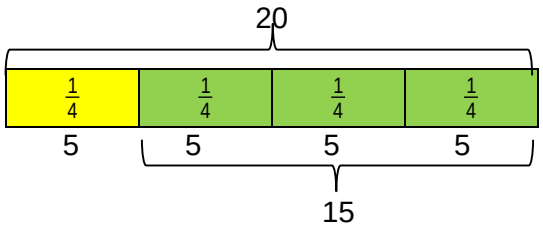
Year 1	• recognise, find and name a half as one of two equal parts of an object, shape or quantity • recognise, find and name a quarter as one of four equal parts of an object, shape or quantity. Part-part whole model using physical objects with 1:1 correspondence with the actual object in the question ↓ Part-part whole model using physical objects with 1:1 correspondence (e.g. different coloured counters, multilink, stones) ↓ Pictorial representation of discrete bar model where each box represents one whole.
	Use cubes and counters to share between a bar split into equal parts. A quarter of 8 8  2 2 2 2 Half of 8 8 



Year 2	• recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity • write simple fractions for example, $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$. Pictorial representation of discrete bar model where each box represents one whole (Year 1). ↓ Concrete representation of continuous bar model where each strip of paper represents a number ↓ Pictorial representation of continuous bar model where each box represents a number.

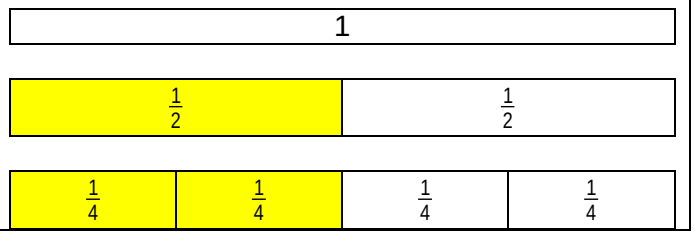
<u>Fractions of amounts</u>

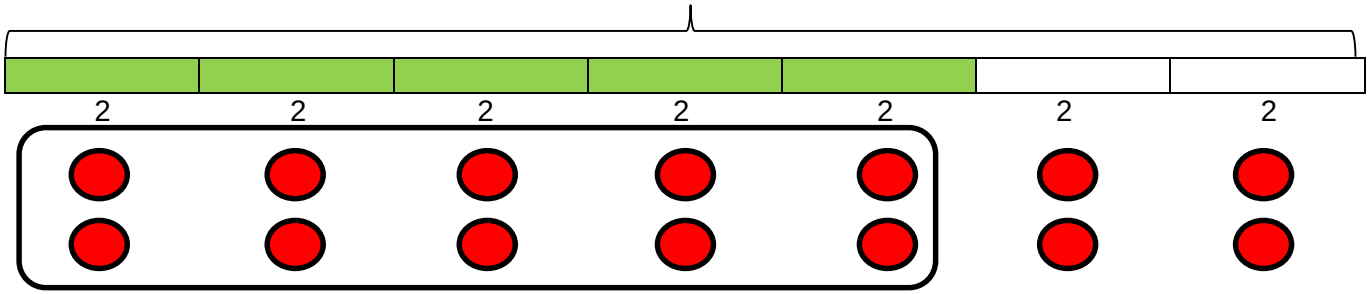
$\frac{1}{4}$ of 20 = 5 $\frac{3}{4}$ of 20 = 15

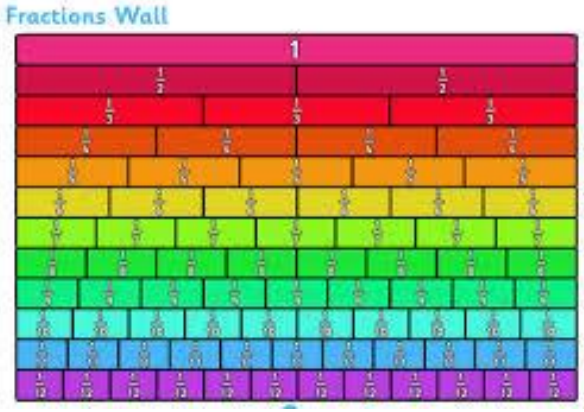


<u>Equivalence</u>

$\frac{2}{4}$ and $\frac{1}{2}$ are equivalent.

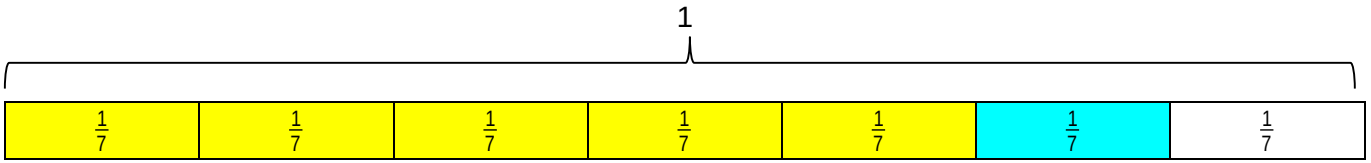


Year 3	• count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10 • recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators • recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators • recognise and show, using diagrams, equivalent fractions with small denominators • add and subtract fractions with the same denominator within one whole [for example, $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$] • compare and order unit fractions, and fractions with the same denominators • solve problems that involve all of the above. Concrete representation of continuous bar model where each strip of paper represents a number ↓ Pictorial representation of continuous bar model where each box represents a number.
	<u>Fractions of amounts – link to use of arrays and use of bar modelling for division</u> $\frac{5}{7}$ of 14 =  14 ÷ 7 = 2, 2 x 5 = 10

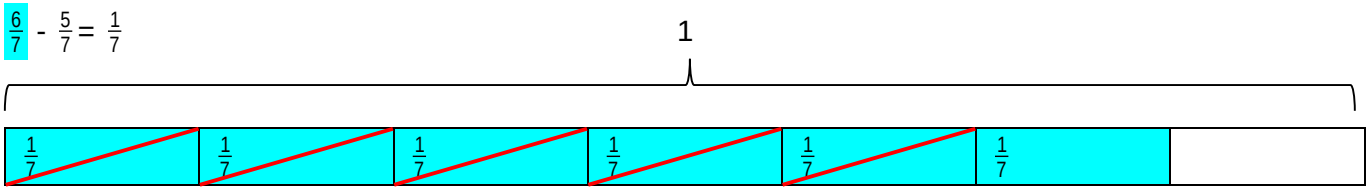
	<u>Equivalent fractions</u> Use a fraction wall or strips to demonstrate how fractions can be equivalent. 

Adding and subtracting fractions

$\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$



$\frac{6}{7} - \frac{5}{7} = \frac{1}{7}$

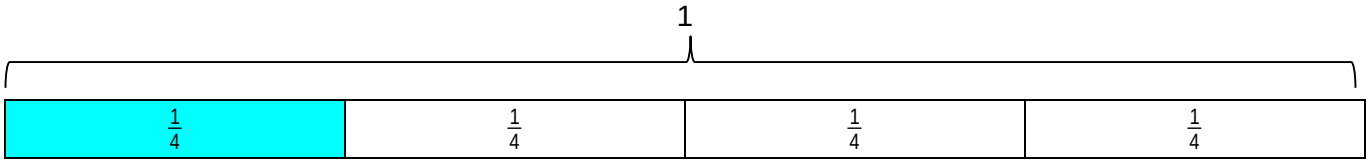


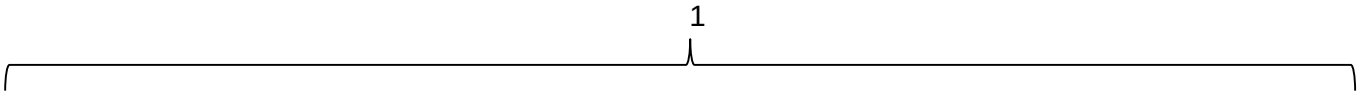
Compare and order fractions

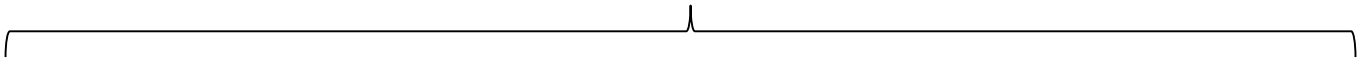
Use/draw a fraction wall.

Same denominator - Ensure when drawing that children are using the squares in their maths book so that the bars are an accurate size.

Which is larger ¼ or ¾

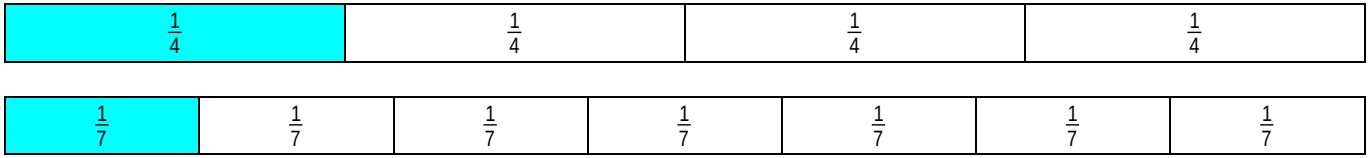






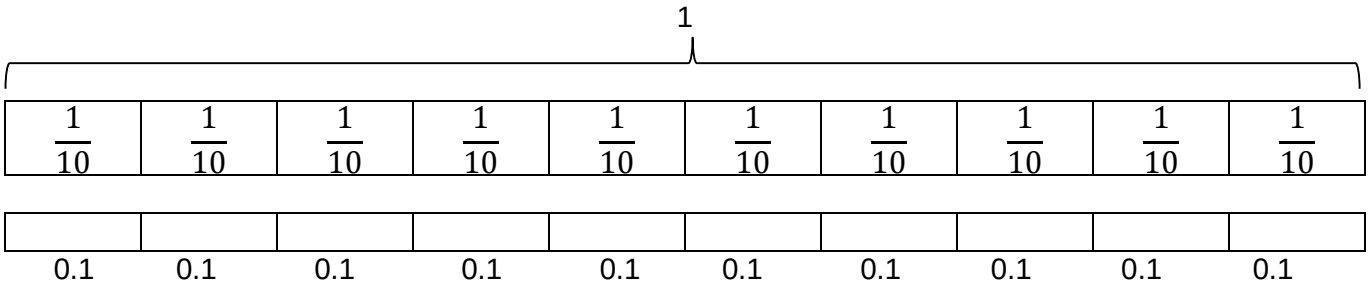
Same numerator – Children should be using plain paper to draw their bar models.

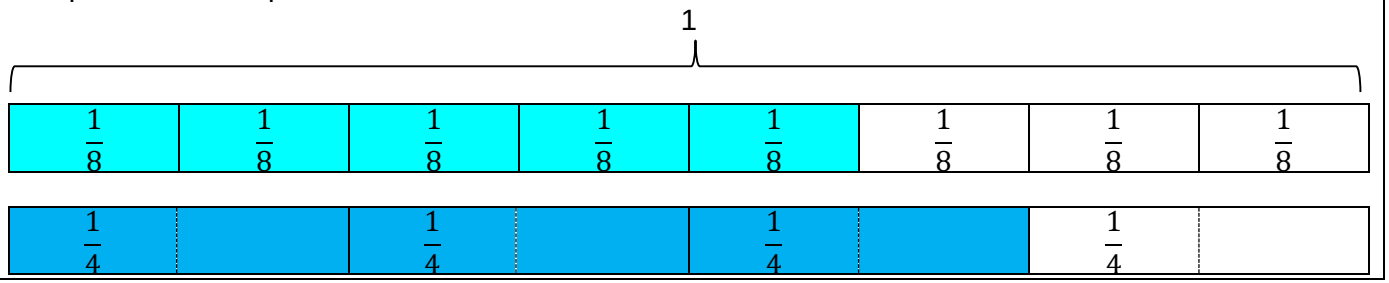
Which is larger: $\frac{1}{4}$ or $\frac{1}{7}$ 1



Fractions on a number line

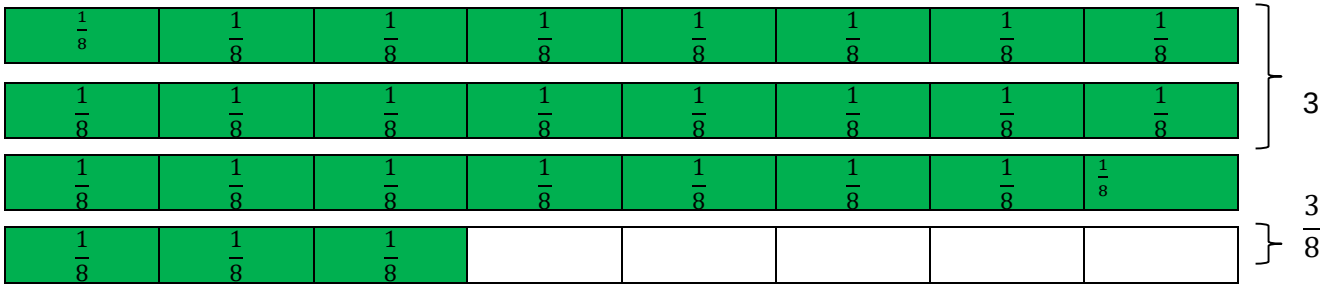
Placing a bar model on top of a number line can help children see the link between bar models and counting.

Year 4	• recognise and show, using diagrams, families of common equivalent fractions • count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten. • solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number • add and subtract fractions with the same denominator • recognise and write decimal equivalents of any number of tenths or hundredths • recognise and write decimal equivalents to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ • find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths • round decimals with one decimal place to the nearest whole number • compare numbers with the same number of decimal places up to two decimal places • solve simple measure and money problems involving fractions and decimals to two decimal places. Concrete representation of continuous bar model where each strip of paper represents a number ↓ Pictorial representation of continuous bar model where each box represents a number.
	<u>Recognise and write decimal equivalents</u> Use an equivalence wall 
	<u>Equivalent Fractions</u> Continued from Year 3
	<u>Adding and Subtracting Fractions</u> Continuation of bar models from Year 3. Most should be able to move to fully mental but should be able to explain why using a bar model if asked.

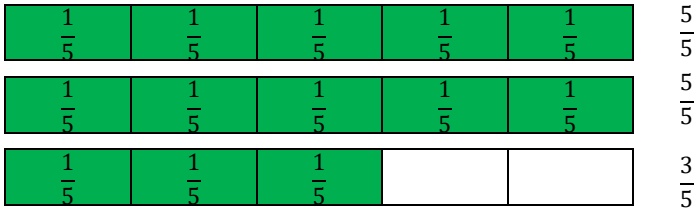
Year 5	• compare and order fractions whose denominators are all multiples of the same number • identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths • recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5}$] • add and subtract fractions with the same denominator and denominators that are multiples of the same number • multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams • read and write decimal numbers as fractions [for example, 0.71 = $\frac{71}{100}$] • recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents • round decimals with two decimal places to the nearest whole number and to one decimal place • read, write, order and compare numbers with up to three decimal places • solve problems involving number up to three decimal places • recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal • solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25. Pictorial representation of continuous bar model where each box represents a number
	<u>Compare and order fractions</u> Which is larger, $\frac{5}{8}$ or $\frac{3}{4}$? Use pictures to compare 

Changing mixed numbers to improper fractions

$\frac{27}{8} = 3\frac{3}{8}$

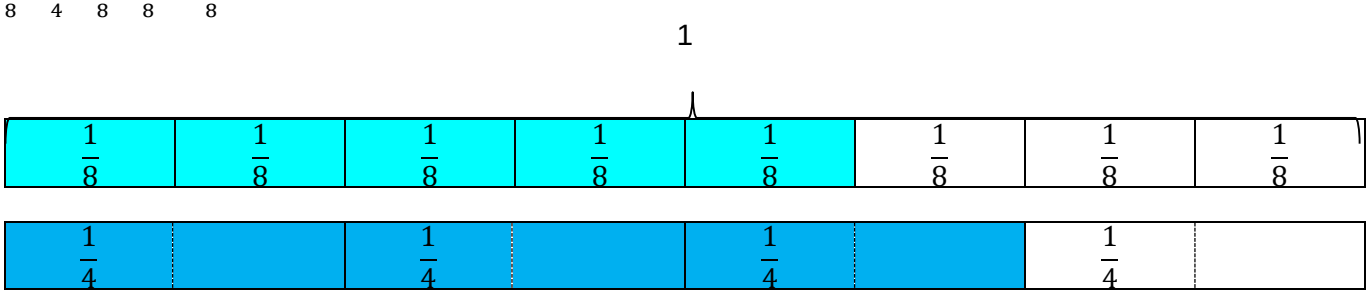


$2\frac{3}{5} = \frac{13}{5}$



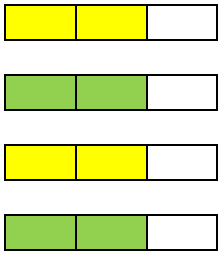
Adding and Subtracting different denominators

$\frac{5}{8} + \frac{3}{4} = \frac{5}{8} + \frac{6}{8} = \frac{11}{8}$

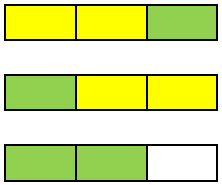


Multiply proper fractions and mixed numbers by whole numbers

$\frac{2}{3}$ x 4 = $\frac{8}{3}$= $2\frac{2}{3}$

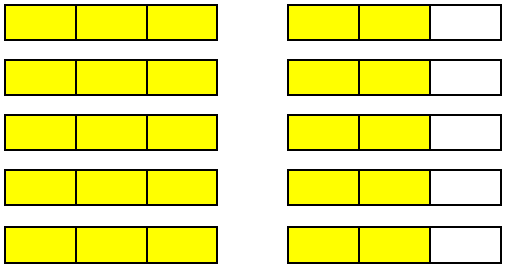


Recombine

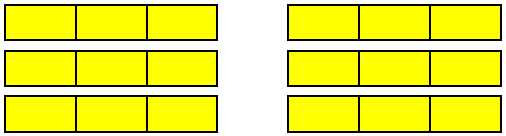


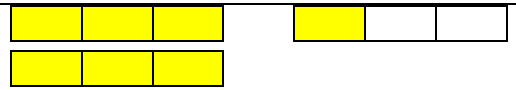
= $2\frac{2}{3}$

5 x $1\frac{2}{3}$ = (5 x 1) + (5 x $\frac{2}{3}$) = 5 + $3\frac{1}{3}$= $8\frac{1}{3}$



Re-combine





$= 8\frac{1}{3}$

Decimal/Percentage/Fraction Equivalence

100%/1 whole

25% ¼ 0.25	25% ¼ 0.25	25% ¼ 0.25	25% ¼ 0.25

100%/1 whole

10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1

Percentages of amounts

25% of 80

100% (80)

25% ¼ 0.25	25% ¼ 0.25	25% ¼ 0.25	25% ¼ 0.25
20	20	20	20

40% of 60 = 24

100% (24)

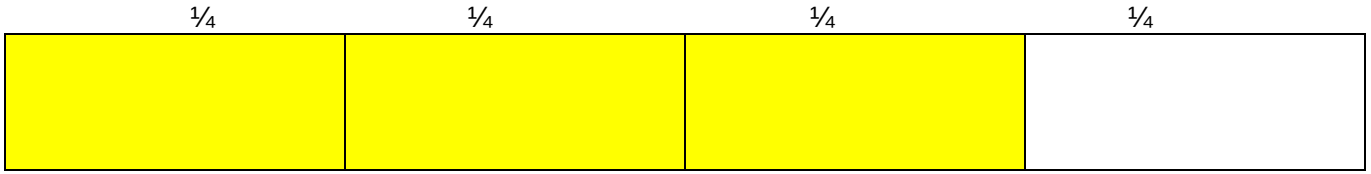
10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1	10% $\frac{1}{10}$ 0.1
6	6	6	6	6	6	6	6	6	6

Year 6	• use common factors to simplify fractions; use common multiples to express fractions in the same denomination • compare and order fractions, including fractions > 1 • add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions • multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$] • divide proper fractions by whole numbers [for example, $\frac{1}{3} \div 2 = \frac{1}{6}$] • associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $\frac{3}{8}$] • identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places • multiply one-digit numbers with up to two decimal places by whole numbers • use written division methods in cases where the answer has up to two decimal places • solve problems which require answers to be rounded to specified degrees of accuracy • recall and use equivalences between simple fractions, decimals and percentages, including in different contexts. Pictorial representation of continuous bar model where each box represents a number.

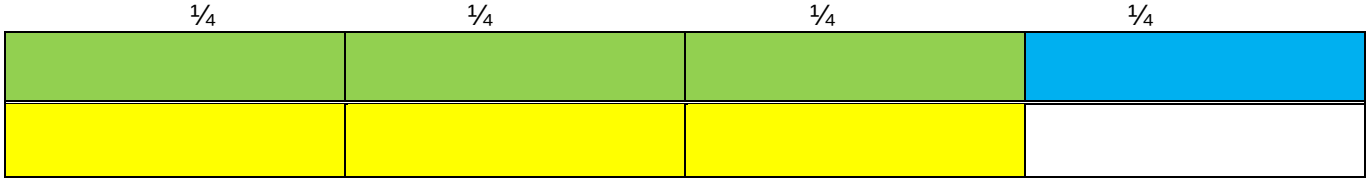
Multiplying proper fractions – NB: It is best to do this on plain paper

$\frac{3}{4} \times \frac{1}{2}$

Draw a bar and split into quarters, then colour ¾



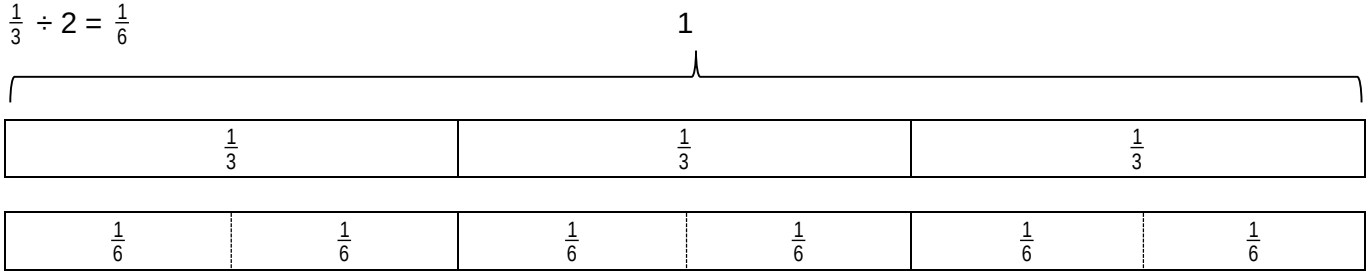
In the other direction, split the bar into halves and colour ½ in a different colour.



The number of boxes that overlap show the fraction. In this case, $\frac{3}{8}$.

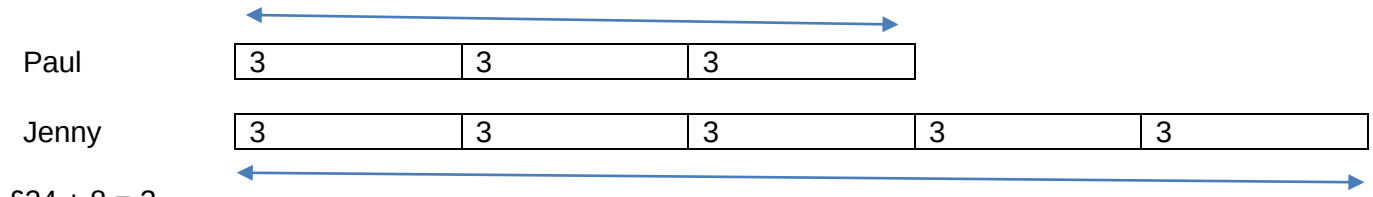
Divide fractions

$\frac{1}{3} \div 2 = \frac{1}{6}$



Ratio

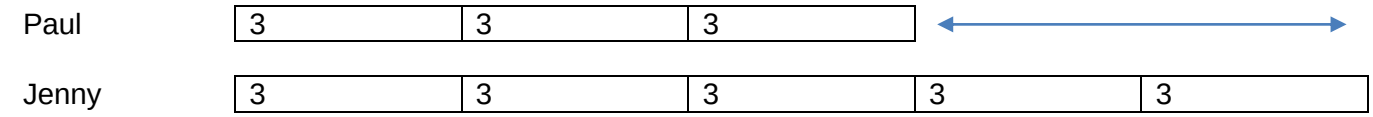
Paul and Jenny share £24 in a ratio of 3:5. How much does Jenny get? How much does Paul get?



£24 ÷ 8 = 3

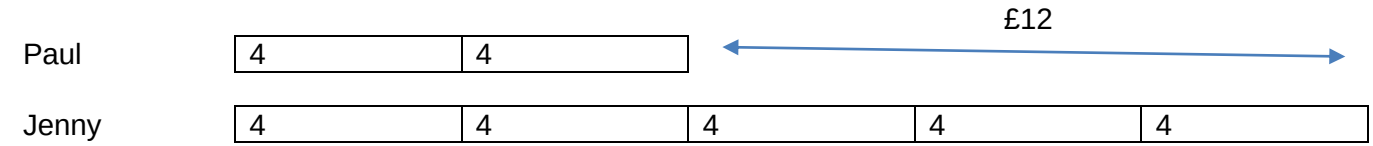
Paul = 3 x 3 = £9
Jenny = 5 x 3 = £15

Jenny and Paul share £24 in a ratio of 5:3. How much more than Paul does Jenny get?

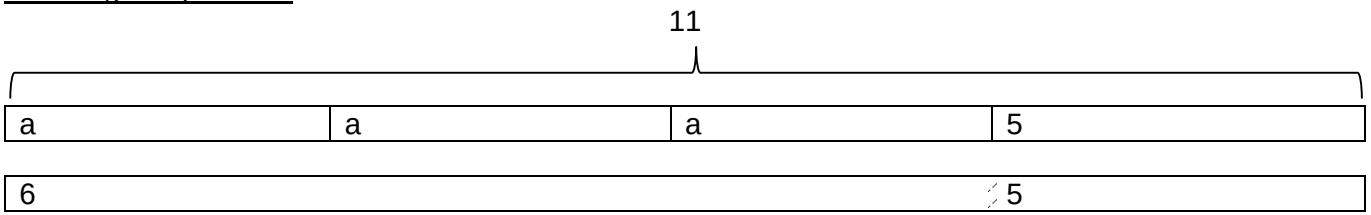


£24 ÷ 8 = £3
3 x 2 = £6

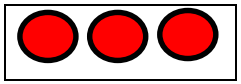
Jenny and Paul share some money in a ratio of 5:2. Jenny gets £12 more than Paul. How much do they share out?

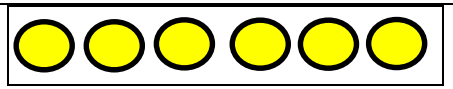


1 part = £12 ÷ 3 = £4
Altogether £4 x 7 = £28

	Algebra 3a + 5 = 11 *Do alongside practical*  3a = 6 a = 6 ÷ 3 = 2 2a + 3 = a + 7  a = 4

Comparison

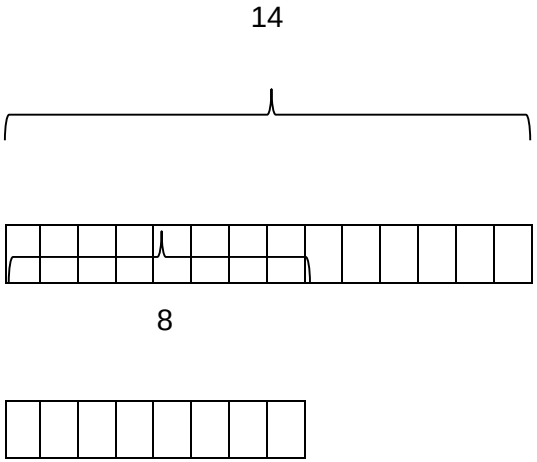
F2	Concrete representation using physical objects with 1:1 correspondence (e.g. different coloured counters, multilink, stones) Part-part whole model using physical objects given in a question (e.g. cars, teddies) and placing in a pre-drawn bar provided by an adult.
	Comparing two numbers. Which is larger: 3 or 6?  3



Year 1	Part-part whole model using physical objects with 1:1 correspondence with the actual object in the question ↓ Part-part whole model using physical objects with 1:1 correspondence (e.g. different coloured counters, multilink, stones) ↓ Pictorial representation of discrete bar model where each box represents one whole.

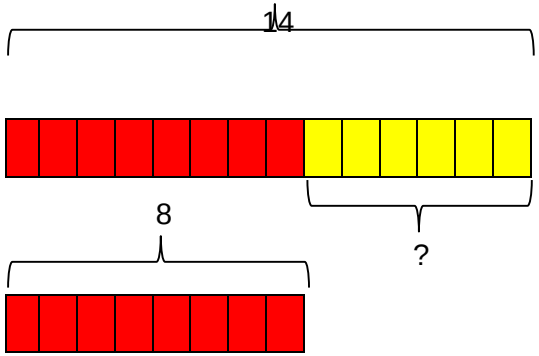
<u>Comparing two numbers.</u>

Which is larger: 14 or 8?



<u>Finding the difference</u>

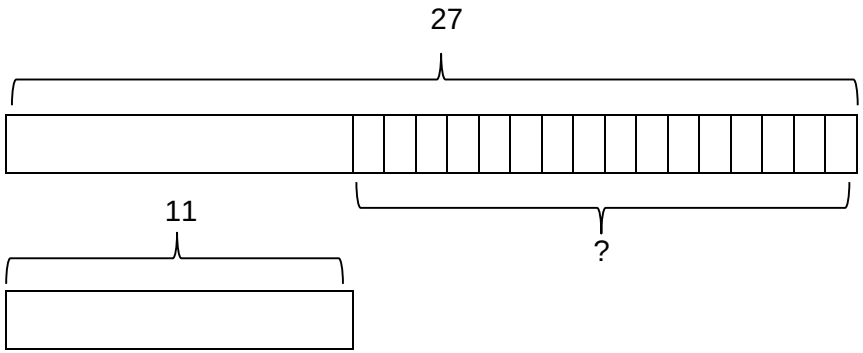
James has 14 pencils and Amy has 8. How many more pencils did Amy have then James?



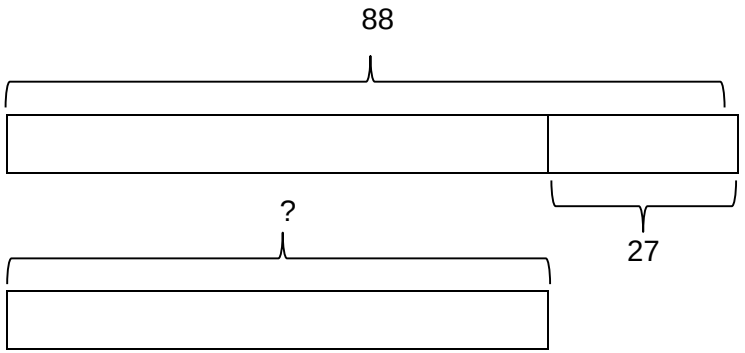
Year 2	Pictorial representation of discrete bar model where each box represents one whole (Year 1). ↓ Concrete representation of continuous bar model where each strip of paper represents a number ↓ Pictorial representation of continuous bar model where each box represents a number.

<u>Finding the difference</u>

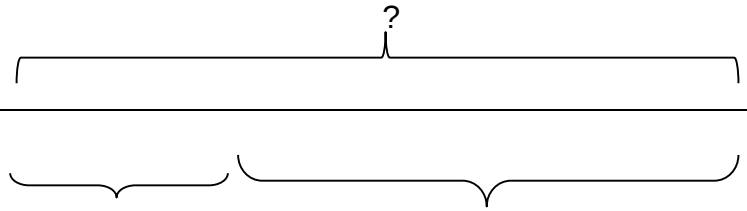
What is the difference between 27 and 11?

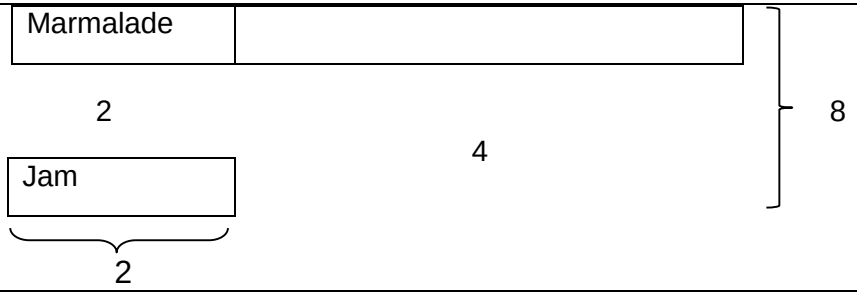


The difference between 2 numbers is 27. The larger number is 8. What is the smaller number?

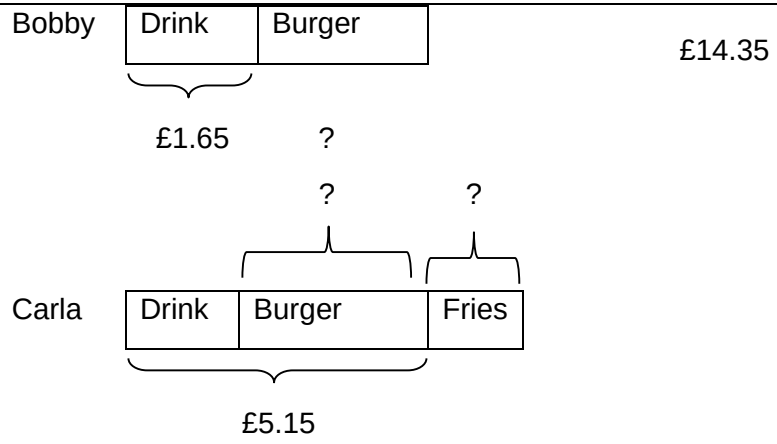


2 people have jam on their toast in the morning. 4 more people have marmalade on their toast than jam. How many people are there altogether?





Years 3 and 4	Concrete representation of continuous bar model where each strip of paper represents a number ↓ Pictorial representation of continuous bar model where each box represents a number.
	<u>Using comparison to find missing quantities</u> Adam, Bobby and Carla went to the café. The total cost of their food was £14.35. Adam had a drink, which cost £1.65, Carla had a drink, burger and fries, and Bobby had a drink and a burger costing £5.15. Find the price of a burger. Find the price of the fries. Draw what you know: £1.65 Adam Drink £5.15



Years 5 and 6	Pictorial representation of continuous bar model where each box represents a number.
	Moving quantities There were two coaches. At first, there were three times as many children on coach A than coach B. 10 people then moved so there was an even number in both coaches. How many were in both coaches altogether?